Study of the $\rho\pi$ Puzzle in Charmonium Decays

MO Xiao-Hu,[*] YUAN Chang-Zheng,[†] and WANG Ping[‡]

Institute of High Energy Physics, CAS, Beijing 100039, China

(Dated: July 2, 2007)

The theoretical explanations about the "$\rho\pi$ puzzle" in charmonium decays are reviewed extensively, and the comparison of theoretical predications with experimental data is made whenever possible. Three methods to estimate the ratio of the branching fractions of J/ψ and ψ' decays are also discussed. It is pointed out that in order to understand the $\rho\pi$ puzzle, and the dynamics of charmonium decays, systematic studies should be made in theory, phenomenology and experiment aspects.

I. INTRODUCTION

Crisply defined experimental puzzles in high-energy physics always have the prospect of leading to new discoveries. One prominent example is the θ-τ puzzle of 1956 which led to the parity revolution. Therefore puzzles in physics often draw considerable attention to theorists. The ratios of hadronic decays of the $\psi(3686)$ (shortened as ψ') to the same decays of the J/ψ is such a puzzle which has been studied substantially since 1983.

Since the OZI suppressed decays of J/ψ and ψ' to hadrons are via three gluons or a photon, in either case, the perturbative QCD (pQCD) provides a relation [1]

$$Q_h = \frac{\mathcal{B}_{\psi'\to h}}{\mathcal{B}_{J/\psi\to h}} = \frac{\mathcal{B}_{\psi'\to e^+e^-}}{\mathcal{B}_{J/\psi\to e^+e^-}} \approx 12.7\% \ . \qquad (1)$$

This relation is referred to as the "12% rule" which is expected to be held to a reasonable good degree for both inclusive and exclusive decays. The so-called "$\rho\pi$ puzzle" is that the prediction by Eq. (1) is severely violated in the $\rho\pi$ and several other decay channels. The first evidence for this effect was found by Mark-II Collaboration in 1983 [2]. From then on many theoretical explanations have been put forth to decipher this puzzle.

With the recent experiment results from BESII and CLEOc about J/ψ and ψ' two-body decays, such as vector-pseudoscalar (VP), vector-tensor (VT), pseudoscalar-pseudoscalar (PP), and baryon-antibaryon ($B\overline{B}$) modes, and about multi-body decays at the J/ψ, the ψ' or even at the $\psi(3770)$ (shortened as ψ'') [3]-[23], a variety of solutions proposed for the puzzle can be tested at the level of higher accuracy. In this treatise, we survey the theoretical works on the $\rho\pi$ puzzle and compare them with the available experimental data. From the theoretical point of view, since the Q-value is smaller than 12% for $\rho\pi$, it may be caused either by enhanced or suppressed J/ψ decay rate. Another possibility is by both. So we classify the relevant theoretical speculations into three categories:

1. J/ψ-enhancement hypothesis, which attributes the small Q-value to the enhanced branching fraction of J/ψ decays.

2. ψ'-suppress hypothesis, which attributes the small Q-value to the suppressed branching fraction of ψ' decays.

3. Other hypotheses, which are not included in the above two categories.

In the following content, first reviewed are the theoretical works on $\rho\pi$ puzzle, and the predictions from them are compared with the newly available experimental results; then expounded are three methods to estimate the ratio between ψ' and J/ψ decays into the same final states; after that some comments are made on the implications from the review on $\rho\pi$ puzzle; at last there is a short summary.

II. REVIEW OF THEORETICAL WORKS ON $\rho\pi$ PUZZLE

A. J/ψ-enhancement Theory

In the earlier days of the $\rho\pi$ puzzle, it was noticed that the decay of 1^{--} charmonium into $\rho\pi$ final state violates the Hadronic Helicity Conservation (HHC) theorem (see below for expound) [24], and so such decay should be suppressed. Therefore people think there must be some mechanism which leads to the great enhancement for $J/\psi \to \rho\pi$ decays. The two schemes presented in this section were proposed following this line of reasoning.

1. J/ψ-glueball Admixture Scheme

The idea of J/ψ decays via a glueball was proposed by Freund and Nambu [25] (FN hereafter) soon after the discovery of J/ψ particle to explain the breaking

———————

[*]Electronic address: moxh@mail.ihep.ac.cn
[†]Electronic address: yuancz@mail.ihep.ac.cn
[‡]Electronic address: wangp@mail.ihep.ac.cn

of Okubo-Zweig-Iizuka (OZI) rule [26]. In such mechanism, the breaking results from the mixing of the ω, ϕ, and J/ψ mesons with an $SU(4)$-singlet vector meson $\mathcal{O}$. They found that such an $\mathcal{O}$ meson should lie in the 1.4-1.8 GeV/c^2 mass range with the width greater than 40 MeV/c^2, and it should decay copiously into $\rho\pi$, $K^*\overline{K}$ while exhibiting severe suppression of decays into $K\overline{K}$, e^+e^- and $\mu^+\mu^-$ modes. These authors presented several quantitative predications for experimental search. Two of them are

$$R_1 = \frac{\Gamma_{J/\psi \to \rho\pi}}{\Gamma_{\phi \to \rho\pi}} = 0.0115 - 0.087 \ ,$$

$$R_2 = \frac{\Gamma_{J/\psi \to K\overline{K}}}{\Gamma_{J/\psi \to \rho\pi}} < 8 \times 10^{-5} \ .$$

With the current available data and using the three pions final state as a substitute for $\rho\pi$ in both ϕ [27] and J/ψ [3, 4] decays, we obtain the first ratio $R_1 \approx 0.003$, which is almost one order of magnitude smaller than the predication. For the second ratio, by virtue of PDG [27] value for K^+K^- and new experiment result for $K^0_S K^0_L$ [6], it is estimated that $\mathcal{B}(J/\psi \to K\overline{K}) \sim 10^{-4}$, together with the results for $\rho\pi$ [3, 4], we have $R_2 \sim 10^{-2}$ which is much larger than the predication.

The first attempt to explain the $\rho\pi$ puzzle in terms of a glueball near J/ψ was proposed by Hou and Soni [28] (HS hereafter). They attributed the enhancement of $J/\psi \to K^*\overline{K}$ and $J/\psi \to \rho\pi$ decay modes to the mixing of the ψ with a $J^{PC} = 1^{--}$ vector gluonium, also designated by $\mathcal{O}$. The differences between FN's and HS's pictures lie in the following aspects:

- Based on the potential model applied to glueball, the mass of a low-lying three-glue state is estimated to be around 2.4 GeV/c^2 [29], rather than 1.4 to 1.8 GeV/c^2 in Ref. [25].

- The mixing of $\mathcal{O}$ with ψ' is taken into account, which has been ignored in previous work.

- Since the gauge coupling constant in QCD is momentum dependent, the mixing parameter is taken to be a function of the invariant mass q^2, which decreases rather sharply with the increase in q^2. Such propagator effect gives rise to a more suppression on the decay rates of ψ' relative to J/ψ for decays to $\rho\pi$ and $K^*\overline{K}$ channels.

By virtue of their assumption, HS suggested a search for the vector gluonium state in certain hadronic decays of the ψ', such as $\psi' \to \pi\pi + X$, $\eta + X$, $\eta' + X$, where X decays into VP final states [28].

Based on HS's idea, Brodsky, Lepage, and Tuan [30] (BLT) refined the glueball hypothesis for the $\rho\pi$ puzzle. They assumed the general validity of the pQCD theorem that the total hadron helicity is conserved in high-momentum-transfer exclusive processes, in which case

the decays to $\rho\pi$ and $K^*\overline{K}$ are forbidden for both the J/ψ and ψ'. This pQCD theorem is often referred to as the rule of Hadronic Helicity Conservation (HHC) [24], which is based on the assumption of the short-range "pointlike" interactions among the constituent quarks throughout. For instance, $J/\psi(c\bar{c}) \to 3g$ has a short range $\simeq 1/m_c$ associated with the short time scale of interaction. Nevertheless, if subsequently the three gluons were to resonate forming an intermediate gluonium state $\mathcal{O}$ which has a large transverse size covering an extended time period, then HHC would become invalid. In essence the HS model takes over in this latter stage.

Final states h which proceed only through the intermediate gluonium state satisfy the ratio

$$Q_h = \frac{\mathcal{B}(\psi' \to e^+e^-)}{\mathcal{B}(J/\psi \to e^+e^-)} \frac{(M_{J/\psi} - M_\mathcal{O})^2 + \Gamma^2_\mathcal{O}/4}{(M_{\psi'} - M_\mathcal{O})^2 + \Gamma^2_\mathcal{O}/4} \ . \quad (2)$$

The Q_h is small if the mass of $\mathcal{O}$ is close to the mass of J/ψ. The experimental limits at that time [2, 30, 31] imply that the $\mathcal{O}$ mass is within 80 MeV/c^2 of the mass of J/ψ and its total width is less than 160 MeV/c^2. Brodsky et $al.$ recommended a direct way to search for $\mathcal{O}$, that is to scan the $e^+e^- \to VP$ cross section across the J/ψ resonance.

Another related work by Chan and Hou [32] studied the mixing angle $\theta_{\mathcal{O}\psi}$ and the mixing amplitude $f_{\mathcal{O}\psi}$ of the J/ψ and vector glueball $\mathcal{O}$ based on the framework of potential models of heavy quarks and constituent gluons. They obtain $|\tan\theta_{\mathcal{O}\psi}| = 0.015$ and $f_{\mathcal{O}\psi}(m^2_{\mathcal{O}\psi}) = 0.008$ GeV^2.

On the experimental part, BES has searched for this hypothetical particle in a $\rho\pi$ scan across the J/ψ mass region in e^+e^- annihilations as well as in the decays of $\psi' \to \pi\pi\mathcal{O}$, $\mathcal{O} \to \rho\pi$, and found no evidence for its existence [8, 33]. The data constrains the mass and width of the $\mathcal{O}$ to the range $| M_\mathcal{O} - M_{J/\psi} | < 80$ MeV/c^2 and $4 < \Gamma_\mathcal{O} < 50$ MeV/c^2 [34]. Although the absence of distortion in BES energy scan of $J/\psi \to \rho\pi$ does not rule out $M_\mathcal{O} \simeq M_{J/\psi}$, it puts a lower bound to $\Gamma_\mathcal{O}$. However, as indicated in Ref. [36], the experimentally constrained mass is several hundred MeV/c^2 lower than the mass of the lightest vector glueball calculated in lattice simulations of QCD without dynamical quarks [37].

Recently, the experimental data from BES and CLEOc turned out to be unfavorable to this glueball hypothesis. Among them is the observed large branching fractions of the isospin-violating VP mode $\psi' \to \omega\pi^0$ [9, 10, 16]. This contradicts with the assertion that the pattern of suppression depends on the spin-parity of the final state mesons. In addition, according to BLT's analysis, one obtains the relation [38]

$$\frac{\mathcal{B}(J/\psi \to \omega\pi^0)}{\mathcal{B}(J/\psi \to \rho^0\pi^0)} < 0.0037$$

which is much smaller than the PDG06 value 0.08 [27]. Another experimental result which is unfavorable to such

hypothesis is the suppression of ψ' decays into vector-tensor (VT) final states [18, 19]. Since hadronic VT decays, unlike the VP decays, conserve HHC, some other mechanisms must be responsible for this suppression in the model. Furthermore, it has been argued that the $\mathcal{O}$ may also explain the decay of J/ψ into ϕf_0 (named previously S^*) but not to $\rho a_0(980)$ (named previously δ), since the $\mathcal{O}$ mixes with the ϕ and enhances a mode that would be otherwise suppressed [30]. However, the decay $\psi' \to \phi f_0$ [34] is not suppressed as found by experiments, which implies the absence of anomalous enhancement in $J/\psi \to \phi f_0$, thus contradicts with this explanation. Anselmino *et al.* extended the idea of J/ψ-$\mathcal{O}$ mixing to the case of $\eta_c \to VV$ and $p\bar{p}$ [39]. They suggested that the enhancement of these decays can be attributed to the presence of a trigluonium pseudoscalar state with a mass close to the η_c mass. So far there is no experimental evidence for such a state.

In fact, as pointed out in Ref. [40], this glueball explanation has some unanswered questions: (i) Why only the $\rho\pi$ and $K^*\overline{K}$ channels are affected but not $\mathcal{O} \to 5\pi$ etc. viz., one must assume in an *ad hoc* way that the $\mathcal{O}$ couples predominantly to $\rho\pi$ and $K^*\overline{K}$; and (ii) if such a narrow heavy 1^{--} gluonium state exists, why have the narrow 0^{++}, 2^{++} states not been found, which should be lighter and easier to search?

2. Intrinsic-charm-component Scheme

Brodsky and Karliner (BK) put forth an explanation for the puzzle based on the existence of the intrinsic charm $|q\bar{q}c\bar{c}\rangle$ Fock components of the light vector mesons [41]. They noticed the fact that quantum fluctuations in a QCD bound state wave function will inevitably produce Fock states containing heavy quark pairs. The intrinsic heavy quark pairs are multiconnected to the valence quarks of the light hadrons, and the wave functions describing these configurations will have maximal amplitude at minimal off-shellness and minimal invariant mass. In the case of the ρ meson, with the consideration of the light-cone Fock representation:

$$\rho^+ = \psi^\rho_{u\bar{d}}|u\bar{d}\rangle + \psi^\rho_{u\bar{d}c\bar{c}}|u\bar{d}c\bar{c}\rangle + \cdots .$$

Here we expect the wave function of the $c\bar{c}$ quarks to be in an S-wave configuration with no nodes in its radial dependence, in order to minimize the kinetic energy of the charm quarks and thus also minimize the total invariant mass.

The presence of the $|u\bar{d}c\bar{c}\rangle$ Fock state in the ρ allows the $J/\psi \to \rho\pi$ decay to proceed through rearrangement of the incoming and outgoing quark lines; in fact, the $|u\bar{d}c\bar{c}\rangle$ Fock state wave function has a good overlap with the radial and spin $|c\bar{c}\rangle$ and $|u\bar{d}\rangle$ wave functions of the J/ψ and pion. On the other hand, the overlap with the ψ' will be suppressed, since the radial wave function of the $n = 2$ quarkonium state is orthogonal to the nodeless $c\bar{c}$ in the $|u\bar{d}c\bar{c}\rangle$ state of the $\rho\pi$. Similarly, the $|u\bar{s}c\bar{c}\rangle$ Fock

component of the K^* favors the $J/\psi K$ configuration, allowing the $J/\psi \to K^*\overline{K}$ decay to also proceed by quark line rearrangement, rather than $c\bar{c}$ annihilation.

These authors also suggested comparing branching fractions for the η_c and η_c' decays as clues to the importance of η_c intrinsic charm excitations in the wave-functions of light hadrons.

B. ψ'-suppress Theory

The hypothesis of the existence of a glueball to explain the $\rho\pi$ has been questioned soon after it was proposed. In addition, it is also pointed out [42] that the helicity suppression is not a strong constraint in the charmonium energy scale. Under such case, one comes naturally to the idea that it is not $J/\psi \to \rho\pi$ which is enhanced, but rather $\psi' \to \rho\pi$ which is suppressed. Seven explanations or models collected in this section are put forth along this line.

1. Sequential-fragmentation Model

Karl and Roberts have suggested explain the $\rho\pi$ puzzle based on the mechanism of sequential quark pair creation [43]. The idea is that the quark-antiquark pairs are produced sequentially, as a result the amplitude to produce two mesons in their ground state is an oscillatory function of the total energy of the system. They argue that the oscillatory fragmentation probability could have a minimum near the mass of ψ', which provides an explanation for the suppressed ψ' decay. Even though their evaluations could generally accommodate the data for decays of J/ψ and ψ' to $\rho\pi$ and $K^*\overline{K}$, it runs into difficulties when it is extrapolated to Υ decays. According to their calculation, the oscillations of probability amplitude are damped out in the region of the Υ resonances, so the $\rho\pi$ channel is present in the decay of all Υ, Υ', Υ'', $\cdots$ resonances with a common rate. This leads to a prediction $\Gamma(\Upsilon \to \rho\pi) = 0.05$ keV, or equivalently $\mathcal{B}(\Upsilon \to \rho\pi) = 9.4 \times 10^{-4}$, which is above the current upper limit $\mathcal{B}(\Upsilon \to \rho\pi) < 2 \times 10^{-4}$ [27]. Moreover, their calculation seems hard to explain the large branching fraction for ϕ decays to $\rho\pi$ [27] due to the fact that their fragmentation probability tends to zero as the mass of the $\rho\pi$ decaying system approaches $1\text{GeV}/c^2$.

In a further analysis [44], Karl and Tuan pointed out that if a suppression is observed in three-meson channels the explanation based on sequential pair creation would be undermined. Recently such a suppressed channel, viz. ϕKK, is found by CLEOc [21].

2. Exponential-form-factor Model

Guided by suppressed ratios of ψ' to J/ψ decays to two-body hadronic modes, Chaichian and Törnqvist sug-

gested [40] that the hadronic form factors fall exponentially as described by the overlap of wave functions within a nonrelativistic quark model. This behavior explains the drastically suppressed two-body decay rates of the ψ' compared with those of the J/ψ. Recently, the report on observation of a number of VP channels in ψ' decays [9, 10, 16] such as $\omega\eta'$, $\phi\eta'$, $\rho\eta'$ has proved that the predicted decay fractions are overestimated. Moreover, the branching fraction for $\omega\pi^0$ [27], is well below the prediction by this model which is 1.04×10^{-4}.

Another problem of the model is that it does not single out just the VP channel, the other channels, for example VT channel, are also estimated to have small branching fractions which are not compatible with the BES measured results [19].

3. Generalized Hindered M1 Transition Model

A so-called generalized hindered M1 transition model is proposed by Pinsky as a solution for the puzzle [45]. It is argued that because $J/\psi \to \gamma\eta$ is an allowed M1 transition while $\psi' \to \gamma\eta'$ is hindered (in the nonrelativistic limit), using the vector-dominance model to relate $\psi' \to \gamma\eta'$ to $\psi' \to \psi\eta'$ one could find the coupling $G_{\psi'\psi\eta_c}$ is much smaller than $G_{\psi\psi\eta_c}$, and then by analogy, the coupling $G_{\omega'\rho\pi}$ would be much smaller than $G_{\omega\rho\pi}$. Here $G_{\omega\rho\pi}$ can be extracted from data by virtue of the analysis using the vector-dominance model and a standard parameterization of OZI process [46]. Then assuming $\psi' \to \rho\pi$ to proceed via ψ'-ω' mixing, while $J/\psi \to \rho\pi$ via ψ-ω mixing, one would find that $\psi' \to \rho\pi$ is much more severely suppressed than $J/\psi \to \rho\pi$. The similar estimation could be preformed for $K^*\bar{K}$ and other VP final state, and one can expect a suppressed Q:

$$\frac{\mathcal{B}(\psi' \to VP)}{\mathcal{B}(\psi \to VP)} = 1.47 \frac{\Gamma_{tot}(\psi)}{\Gamma_{tot}(\psi')} \left(\frac{G_{V'VP}}{G_{VVP}}\right)^2 \frac{F_{V'}}{F_V} = 0.06\% ,$$
(3)

where $F_{V'}/F_V = 0.3$, $G_{\omega'\rho\pi}/G_{\omega\rho\pi} = 0.066$ according to Ref. [45]. This Q is much smaller than the present experimental results [9, 10, 16].

Moreover, in this model, the coupling $G_{\omega'\omega f_2}$ for $\omega' \to \omega f_2$ should not be suppressed because by analogy the coupling $G_{\psi'\psi\chi_{c2}}$ is not small due to the fact that the E1 transition $\psi' \to \gamma\chi_{c2}$ is not hindered [47]. Therefore via ψ'-ω' mixing the $\psi' \to \omega' \to \omega f_2$ decay is expected to be not suppressed, which contradicts the BES result [19].

4. Higher-Fock-state Scheme

Chen and Braaten (CB) proposed an explanation [36] for the $\rho\pi$ puzzle, arguing that the decay $J/\psi \to \rho\pi$ is dominated by a Fock state in which the $c\bar{c}$ pair is in a color-octet $^3S_1$ state which decays via $c\bar{c} \to q\bar{q}$, while the suppression of this decay mode for the ψ' is attributed to a dynamical effect due to the small energy gap between the mass of the ψ' and the $D\bar{D}$ threshold. Using the BES data on the branching fractions into $\rho\pi$ and $K^*\bar{K}$ as input, they predicted the branching fractions for many other VP decay modes of the ψ', as listed in Table I, from which we see most measured values falling in the scopes of predictions, but we also note for $\omega\pi$ mode, the deviation from the prediction is obvious. Here it should be noticed that the values adduced in Table I are calculated on the strength of the measured branching fractions from earlier experiment, the new measurements on the branching fractions for $\rho\pi$ and $K^{*0}\bar{K}^0 + c.c.$ from BES [11, 14] and CLEOc [16] may have impact on the corresponding evaluations.

TABLE I: Predictions and measurements for Q_{VP} in unit of 1% for all VP final states. The value for $\rho\pi$ and $K^{*0}\bar{K}^0 + c.c.$ from Ref. [48] were used as input. The theoretical parameter $x = 0.64$ is due to results of Ref. [49] and the experimental results come from Ref. [9–11, 50].

VP	$x = 0.64$	Exp.
$\rho\pi$	$0 - 0.25$	0.13 ± 0.03
$K^{*0}\bar{K}^0 + c.c.$	$1.2 - 3.0$	3.2 ± 0.08
$K^{*+}K^- + c.c.$	$0 - 0.36$	$0.59^{+0.27}_{-0.36}$
$\omega\eta$	$0 - 1.6$	< 2.0
$\omega\eta'$	$12 - 55$	19^{+15}_{-13}
$\phi\eta$	$0.4 - 3.0$	5.1 ± 1.9
$\phi\eta'$	$0.5 - 2.2$	9.4 ± 4.8
$\rho\eta$	$14 - 22$	$9.2^{+3.6}_{-3.3}$
$\rho\eta'$	$12 - 20$	$17.8^{+15.9}_{-11.1}$
$\omega\pi$	$11 - 17$	$4.4^{+1.9}_{-1.6}$

Besides the predictions in Table I, CB's proposal also has implications for the angular distributions for two-body decay modes. In general, the angular distribution has the form $1 + \alpha\cos^2\theta$, with $-1 < \alpha < +1$. CB's conclusion implies that the parameter α for any two-body decay of the ψ' should be less than or equal to that for the corresponding J/ψ decays. But this needs further supports from analyses based on large data sample in the future.

5. Survival-chamonia-amplitude Explanation

A model put forward by Gérard and Weyers entertains the assumption that the three-gluon annihilation amplitude and the QED amplitude add incoherently in all channels in J/ψ decays into light hadrons, while in the case of ψ' decays the dominant QCD annihilation amplitude is not into three gluons, but into a specific configuration of five gluons [51]. More precisely, they suggest that the strong annihilation of the ψ' into light hadrons is a two-step process: in the first step the ψ' goes into two gluons in a 0^{++} or 0^{-+} state and an off-shell $h_c(3526)$; in the second step the off-shell h_c annihilates into three gluons to produce light hadrons. Their argument implies:

(a) to leading order there is no strong decay amplitude for the processes $\psi' \to \rho\pi$ and $\psi' \to K^*\overline{K}$; (b) the 12 % rule should hold for hadronic processes which take place via the QED amplitude only. As far as the second implication is concerned, the present data give different ratios between ψ' and J/ψ decay for $\omega\pi^0$ and $\pi^+\pi^-$ final states, both of which are electromagnetic processes. Here the event form factor effect is taken into account [52], the difference between two kinds of processes is still obvious. Besides the explanation for $\rho\pi$ puzzle, this model predicts a sizable $\psi' \to (\pi^+\pi^-$ or $\eta)\, h_1(1170)$ branching fraction.

In a recent paper [53], Artoisenet, Gérard and Weyers (AGW) update and sharpen the above idea which leads to a somewhat unconventional point of view: all non-electromagnetic hadronic decays of the ψ' goes via a transition amplitude which contain a $c\bar{c}$ pair. AGW provide two patterns for these two-step decays, the first is

$$\psi' \to (2\text{NP}g) + (3g) . \tag{4}$$

The physics picture is as follows: the excited $c\bar{c}$ pair in the ψ' does not annihilate directly. Instead, it spits out two non-perturbative gluons $(2\text{NP}g)$ and survives in a lower $c\bar{c}$ configuration $(1^{--}$ or $1^{-+})$ which then eventually annihilate into $3g$. The decays $\psi' \to (2\pi)J/\psi$ and $\psi' \to \eta J/\psi$ follow this pattern. The second pattern is

$$\psi' \to (3\text{NP}g) + (2g) , \tag{5}$$

where the lower $c\bar{c}$ configuration $(0^{-+}$ or $0^{++})$ annihilates into $2g$. The only on-shell channel for this type of decays is $\psi' \to (3\pi)\eta_c$, whose branching fraction is estimated as $(1-2)\%$ level. Anyway, the recent measurement from CLEOc [20] provides the upper limit which is one order of magnitude below this theoretical prediction. Furthermore, the substitution of one photon for one gluon in Eqs. (4) and (5) allows

$$\psi' \to (2\text{NP}g) + (2g) + \gamma . \tag{6}$$

This pattern corresponds to on-shell radiative decays such as $\psi' \to (\pi^+\pi^-)\eta_c\gamma$ and $\psi' \to \eta\eta_c\gamma$, which could be larger than the observed $\psi' \to \eta_c\gamma$ mode.

Besides the above predications, AGW also estimate

$$\mathcal{B}(\psi' \to b_1\eta) = (1.3 \pm 0.3) \times 10^{-3} , \tag{7}$$
$$\mathcal{B}(\psi' \to h_1\pi^0) = (1.9 \pm 0.4) \times 10^{-3} , \tag{8}$$
$$\mathcal{B}(J/\psi \to b_1\eta) \approx \mathcal{B}(\psi' \to b_1\eta) \approx 1\% . \tag{9}$$

All these are to be tested by experiments.

6. Nonvalence Component Explanation

Since the ψ' is a highly excited state and close to the $D\overline{D}$ threshold, it is suggested [54] that unlike the J/ψ, the ψ' may be an admixture of large nonvalence components in its wave function. The so-called nonvalence component indicates the additional gluon or the light quark-antiquark pair (or as in Ref. [54], the $c\bar{c}g$ component and a quasi-molecular $D\overline{D}$ state), which makes ψ' decays prominently distinctive from those of J/ψ. The nonvalence component of the J/ψ is expected to be less significant because it is the lowest state. In a following paper [42], Chernyak uses this picture to explain the $\rho\pi$ puzzle. He suggested that the valence and nonvalence strong contributions interfere destructively in $\rho\pi$ channel and consequently cancel to a large extent in the total $\psi' \to \rho\pi$ strong amplitude, while the role of nonvalence contributions is much less significant in $J/\psi \to \rho\pi$. From this viewpoint, there is no deep reason for the experimentally observed very strong suppression of $\psi' \to \rho\pi$, this is the result of a casual cancellation.

Chernyak also tries to use the above idea to explain qualitatively other decay modes, such as VT, AP, PP, VV and $B\overline{B}$ decay. However, such an idea remains a pure speculation, and no concrete calculations have been performed up to now.

7. S-D Wave Mixing Scheme

The ψ'' is viewed as a D-wave charmonium state. Although it is primarily $c\bar{c}(1^3D_1)$, its leptonic width indicates a contribution from mixing with S-wave states, mainly the nearby $\psi(2^3S_1)$. This leads Rosner to propose that the small $\rho\pi$ branching fraction in ψ' decay is due to the cancellation of the $2S$ and $1D$ matrix elements. By virtue of his scheme

$$\begin{aligned} \langle\rho\pi|\psi'\rangle &= \langle\rho\pi|2^3S_1\rangle \cos\theta - \langle\rho\pi|1^3D_1\rangle \sin\theta , \\ \langle\rho\pi|\psi''\rangle &= \langle\rho\pi|2^3S_1\rangle \sin\theta + \langle\rho\pi|1^3D_1\rangle \cos\theta , \end{aligned} \tag{10}$$

where θ is the mixing angle between pure $\psi(2^3S_1)$ and $\psi(1^3D_1)$ states [55] and is fitted from the leptonic widths of the ψ'' and the ψ' to be $(12 \pm 2)°$ [56], which is consistent with the coupled channel estimates [57, 58] and with the ratio of ψ' and ψ'' partial widths to $J/\psi\pi^+\pi^-$ [59]. If the mixing and coupling of the ψ' and ψ'' lead to complete cancellation of $\psi' \to \rho\pi$ decay $(\langle\rho\pi|\psi'\rangle = 0)$, the missing $\rho\pi$ decay mode of the ψ' shows up instead as decay mode of the ψ'', enhanced by the factor $1/\sin^2\theta$, the concrete estimation shows that [56]

$$\mathcal{B}_{\psi''\to\rho\pi} = (4.1 \pm 1.4) \times 10^{-4} . \tag{11}$$

To test this scenario by the data collected at the ψ'' in e^+e^- experiments, it has been pointed out [60, 61] that the continuum contribution must be considered carefully. Specifically speaking, by Rosner's estimation, the Born order cross section for $\psi'' \to \rho\pi$ is

$$\sigma^{Born}_{\psi''\to\rho\pi} = (4.8 \pm 1.9) \text{ pb} , \tag{12}$$

which is comparable in magnitude to that of the continuum cross section, viz.

$$\sigma^{Born}_{e^+e^-\to\rho\pi} = 4.4 \text{ pb} . \tag{13}$$

So what is observed is the coherent sum of the two amplitudes. Based on the analysis of experimental data, it has been suggested that there be a universal phase between strong and electromagnetic amplitudes in charmonium decays. With this assumption, the strong decay amplitude interferes with the continuum amplitude either maximum destructively, e.g. for $\rho\pi$, $\omega\eta$ and $K^{*+}K^-$ or maximum constructively, e.g. for $K^{*0}\overline{K^0}$. The destructive interference leads to the phenomena that the measured cross section on top of the resonance could be smaller than that off the resonance. Recent experimental results [13, 22] on $\rho\pi$, $\omega\eta$ and $K^{*+}K^-$ have demonstrated such interference pattern. This provides support to the prediction of Eq.(11). However, to uniquely determine $\mathcal{B}_{\psi''\to\rho\pi}$, current available experimental data are insufficient, the ψ'' resonance must be scanned [62]. So the quantitative test of Rosner's scenario remains to be the task of the future experiments on τ-Charm factories.

In the consequent study [63], Wang, Yuan and Mo (WYM) extend the S-D wave-mixing scenario to PP final state, and give a constraint for $\psi''\to K_S^0 K_L^0$ decay,

$$0.12\pm0.07 \leq 10^5 \times \mathcal{B}(\psi''\to K_S^0 K_L^0) \leq 3.8\pm1.1 \ , \quad (14)$$

which is compatible with both the BESII result $\mathcal{B}(\psi''\to K_S^0 K_L^0) < 2.1\times10^{-4}$ at 90% C.L. [12] and the CLEOc result $\mathcal{B}(\psi''\to K_S^0 K_L^0) < 1.17\times10^{-5}$ at 90% C.L. [23]. Extrapolating fore-study to all charmless decays [64], WYM found that this scenario could accommodate large non-$D\overline{D}$ decay of the ψ''. Although the recent experimental measurement from CLEOc favors the zero non-$D\overline{D}$ cross section at ψ'' [65], the larger error could not rule out the existence of non-$D\overline{D}$ branching fraction at 10% level.

C. Other Explanations

Besides the models in two previous sections, more speculations involving the $\rho\pi$ puzzle are collected in this section.

1. Final State Interaction Scheme

Li, Bugg and Zou [66] (LBZ) pointed out that the final state interactions (FSI) in J/ψ and ψ' decays give rise to effects which are of the same order as the tree level amplitudes, they argued that $J/\psi\to\rho\pi$ is strongly enhanced by the $a_2\rho$ loop diagram, while the direct tree-production for $\rho\pi$ may be suppressed by the HHC mechanism [24]. The contribution of the $a_1\rho$ loop diagram is much smaller than that of $a_2\rho$ loop for the $J/\psi\to\rho\pi$, but they have similar strength for the $\psi'\to\rho\pi$ and may cancel each other by interfering destructively. The similar apparent suppression for $\psi'\to K^*\overline{K}$ and $f_2\omega$ may also be explained by the $K^*\overline{K}^*_{2,1}$ and $b_1\pi$ loop, respectively. Therefore, LBZ expected that FSI may provide a coherent explanation for all the observed suppressed modes of

ψ' decays. However, as remarked in Ref. [67], this interference model does appear to have more assumptions that predictions. The only qualitative prediction by LBZ is to check whether the $a_1\rho$ and $K_1^*\overline{K}^*$ production rates are large for the ψ'. So far, no such measurements have been reported. Nevertheless, useful information on $a_1\rho$ and $K_1^*\overline{K}^*$ could be obtained from the BES published data as shown in Refs. [17] and [19]. The lack of evidence within the invariant mass distribution plots (see Fig.3 and Fig.5 of Ref. [17] or Ref. [19]) that the $\rho\pi$ recoiled against a ρ for events of $\psi'\to\rho^0\rho^\pm\pi^\mp$ and that $\pi^\pm K^\mp$ recoiled against a K^{*0} for events of $\psi'\to\pi^+\pi^- K^+K^-$ suggests that they are unlikely to be the favored modes in ψ' decays.

2. Large Phase Scheme

Suzuki gave another explanation based on FSI for J/ψ decays [38]. He performed a detailed amplitude analysis for $J/\psi\to 1^-0^-$ decay to test whether or not the short-distance FSI dominates over the long-distance FSI in the J/ψ decay. His result indicates that there is a large phase between three-gluon and one-photon amplitudes. Since the large phase cannot be produced with the perturbative QCD interaction, the source of it must be in the long-distance part of strong interaction, namely, rescattering among hadrons in their inelastic energy region. Suzuki then performed the similar analysis for $J/\psi\to 0^-0^-$ decay, and obtained the same conclusion of the large phase [68]. His analysis also shows that the exclusive decay rate at the J/ψ is in line with that of the inclusive decay. This fact leads him to believe that the origin of the relative suppression of $\psi'\to 1^-0^-$ to $J/\psi\to 1^-0^-$ is not in the J/ψ but in the ψ'.

As to this large phase, in Ref. [51], Gérard argued that this phase follows from the orthogonality of three-gluon and one-photon virtual processes. As a matter of fact, the conclusion of a large phase close to 90° between three-gluon and one-photon processes holds true for all two-body decays of J/ψ, such as 1^+0^- (90°) [69], 1^-0^- $(106\pm10)°$ [70, 71], 1^-1^- $(138\pm37)°$ [68, 71, 72], 0^-0^- $(89.6\pm9.9)°$ [68, 71, 72] and $N\overline{N}$ $(89\pm15)°$ [71, 73].

Nevertheless, as the ψ' decays were analyzed, the experimental data at first seemed to favor a phase close to 180° [69], in contrary to the expectation that the decay of J/ψ and ψ' should not be much different. However, as pointed out by Wang *et al.* that the previous published data did not take the continuum one-photon process into account. Their reanalysis of the experimental data shows that the phase with value around $-90°$ could fit $\psi'\to 1^-0^-$ data [61] and $\pm90°$ could fit $\psi'\to 0^-0^-$ data [74]. The latter is confirmed by more recent results by CLEO [5]. Furthermore, this large phase also shows in the OZI suppressed decay modes of the ψ''. In many decays modes of the ψ'', the strong decay amplitudes have comparable strength as the non-resonance continuum amplitude, the large phase around $-90°$ leads

to destructive or constructive interference. Due to the destructive interference, the observed cross sections of some modes at the peak of the ψ'' are smaller than the cross section measured off-resonance [60]. This is demonstrated by the data from CLEOc [22].

If the large phase between the three-gluon and one-photon virtual processes is universal not only in J/ψ decays, but also in ψ' as well as all charmonium or perhaps all quarkonium decays, then how this phase is related to the difference between the decays of J/ψ and ψ' remains to be an unanswered question.

3. Mass Reduction Explanation

In the study [75] of radiative decays of 1^{--} quarkonium into η and η', Ma presented a QCD-factorization approach, with which he obtained theoretical predictions in consistency with CLEOc measurement. The largest possible uncertainties in analysis are from the relativistic corrections for the value of the charm quark mass. Ma argued that the effect of these uncertainties can be reduced by using quarkonium masses instead of using quark mass. As an example of such reduction, he provided a modified relation to the original 12% rule

$$Q_{\rho\pi} = \frac{\mathcal{B}(J/\psi \to \rho\pi)}{\mathcal{B}(\psi' \to \rho\pi)} = \frac{M_{J/\psi}^8}{M_{\psi'}^8} \frac{\mathcal{B}(J/\psi \to e^+e^-)}{\mathcal{B}(\psi' \to e^+e^-)}$$

$$= (3.6 \pm 0.6)\% \ .$$

However, this value is much larger than the experimental result given in Table I.

4. Vector-meson-mixing Model

Intending to give a comprehensive description of J/ψ two-body decays, Clavelli and Intemann (CI) proposed a vector-meson-mixing model in which the vector mesons $(\rho, \omega, \phi, J/\psi)$ are regarded as being admixture of light-quark-antiquark state and charmed-quark-antiquark state [76]. The coupling of the J/ψ to any state of light quarks is then related to the corresponding coupling of the ρ, ω, and ϕ to the same state. With few experiment inputs to determine the mixing parameters, CI calculated VP, PP, and BB decay rates for the J/ψ as a function of the pseudoscalar mixing angle. Most of the predictions agree with the experiment results at the order of magnitude level, but discrepancy is obvious for some channels, such as $K_S^0 K_L^0$ final state [6]. CI also extended their model to the hadronic decays of the ψ'. Nevertheless, their evaluations for $\mathcal{B}(J/\psi \to \omega\pi^0) = 3 \times 10^{-5}$ and $\mathcal{B}(\psi' \to \omega\pi^0) = 3 \times 10^{-3}$ contradict with the present results $(4.5 \pm 0.5) \times 10^{-4}$ and $(2.1 \pm 0.6) \times 10^{-4}$ [27], respectively.

Starting from effective Lagrangian whereby nonet-symmetry breaking and pseudoscalar-meson mixing can

be studied, Haber and Perrier parametrized the decay modes of $J/\psi \to PP$ (also for $J/\psi \to VV$ or $\eta_c \to VP$), $J/\psi \to VP$ (also for $J/\psi \to VT$ or $\eta_c \to VV$), $J/\psi \to PPP$ (also for $J/\psi \to VVP$ or $\eta_c \to PPV$), and $\eta_c \to PPP$ (also for $J/\psi \to PPV$ or $\eta_c \to VVP$) [77]. The experimental data are used to determine these phenomenological parameters. In a later work, Seiden, Sadrozinski and Haber took the doubly Okubo-Zweig-Iizuka suppression (DOZI) effect into consideration, and presented a more general parameterization of amplitudes for $J/\psi \to PP$ decays [78]. With this form, one could easily derive the relative decay strength between different final states. However, it has also been noticed that under the most general circumstances, symmetry breaking introduces too many parameters for a general analysis to be useful. In order to reduce the number of new parameters considerably and make the analysis manageable, only special cases could be considered.

TABLE II: Comparison of predictions [79] and measurements [27] for the branching ratios (10^{-5}) for ψ' decays. The upper limits are presented at 90% C.L.

VP	Prediction	Measurement
$\rho\pi$	1.3	3.2 ± 1.2
$K^{*0}\overline{K^0} + c.c.$	5.1	10.9 ± 2.0
$K^{*+}K^- + c.c.$	1.3	$1.7^{+0.8}_{-0.7}$
$\omega\eta$	1.2	< 1.1
$\omega\eta'$	6.3	$3.2^{+2.5}_{-2.1}$
$\phi\eta$	1.6	$2.8^{+1.0}_{-0.8}$
$\phi\eta'$	4.6	3.1 ± 1.6
$\rho\eta$	2.1	2.2 ± 0.6
$\rho\eta'$	1.2	$1.9^{+1.7}_{-1.6}$
$\omega\pi^0$	3.8	2.1 ± 0.6
$\phi\pi^0$	0.01	< 0.40

A similar parameterization with mixing feature of the strong interaction mechanism was proposed by Feldmann and Kroll (FK) [79] for the hadron-helicity nonconserving J/ψ and ψ' decays, but with a different interpretation from those put forth in Refs. [36, 49, 67, 78]. FK assumed that with a small probability, the charmonium possesses Fock components built from light quarks only. Through these Fock components the charmonium state decays by a soft mechanism which is modeled by J/ψ-ω-ϕ mixing and subsequent ω (or ϕ) decays into the VP state. In absence of the leading-twist perturbative QCD contribution, the dominant mechanism is supplemented by the electromagnetic decay contribution and DOZI violating contribution. FK argued that this mechanism can probe the charmonium wave function at all quark-antiquark separations and feels the difference between a $1S$ and a $2S$ radial wave functions. The node in the latter is supposed to lead to a strong suppression of the mechanism in ψ' decays. With a few parameters adjusted to the experimental data, FK obtained a numerical description of the branching fractions for many VP decay modes of the J/ψ and ψ', which agree with the

measured branching fractions at the order of magnitude level, as shown in Table II. Moreover, FK has extended their mixing approach to the $\eta_c \to VV$ decays and obtained a reasonable description of the branching fractions for these decays while the $\eta_c' \to VV$ decays are expected to be strongly suppressed.

D. Remarks

From the brief retrospect of $\rho\pi$ puzzle history, we could notice that in the early stage, theorists concentrate on peculiar mechanism for special channels like $\rho\pi$ and $K^*\overline{K}$, such as Hou and Soni's explanation. With the development of the theory and the progress of the experiment, theorists attempt to provide a more general scheme for charmonium decays, such as the work by Feldmann and Kroll. In fact, the charmonium decay is an interconnected system as a whole, a correct explanation of $\rho\pi$ puzzle is expected to describe, quantitatively, or semi-quantitatively, the properties of all measured decay modes.

At last, a few words about Fock state component. This concept has been discussed in many works, such as by BK [41], CB [36], and FK [79]. Even from the theoretical explanation of experiments other than e^+e^- collision, the non-perturbative Fock component is also indicated to exist. For example, the analysis of charmonia photoproduction amplitude implies [80] that the $|c\bar{c}\rangle$ component is narrowly distributed in the transversal direction, while the $c\bar{c}$ pairs at larger separations may be part of higher Fock states which contain gluons and light quarks.

III. EVALUATION OF RATIO OF ψ' TO J/ψ DECAY

In this section, we depict three approaches to estimate the ratio of ψ' to J/ψ decay, one from theoretical analysis [1], the other two from experimental evaluation [81].

A. Theoretical Method

Conventionally, the measured ratios of ψ' to J/ψ branching fractions for specific exclusive hadronic decays are compared with the naive prediction of pQCD, the so-called "12% rule". In the framework of pQCD [1], the ψ particles are considered to be nonrelativistic bound states of a charm quark and its antiquark. Their decays into light hadrons are believed to be dominated by the annihilation of the $c\bar{c}$ pair into three gluons. In order to annihilate, the c and $\bar{c}$ must have a separation of order $1/m_c$, which is much smaller than the size of the charmonium state. Thus the annihilation amplitude for an S-wave state like J/ψ and ψ' must be proportional to the wave function at the origin, $\psi(\mathbf{r} = 0)$ [1]. The

width for the decay into any specific final state h consisting of light hadrons is therefore proportional to $|\psi(0)|^2$. The width for the decay into e^+e^- is also proportional to $|\psi(0)|^2$. This leads to the simple prediction that the ratio of the branching fractions between ψ' and J/ψ is given by Eq. (1).

However, this naive prediction suffers from several apparent approximations. Higher order corrections, which may not even be small, are not included in this calculation. For example, a first order correction to the branching fraction of $J/\psi \to e^+e^-$ could be 50% of the lowest term if $\alpha_s(m_{J/\psi}) \sim 0.2$ [82] is taken. The relativistic effect is also ignored. Since the mass difference between J/ψ and ψ' is around 20% and $< v^2/c^2 > \sim 0.24$ for J/ψ, this correction may be at the same level as the lowest order contribution [82]. The inclusion of the finite size of the decay vertex will significantly reduce the ggg decay width of the J/ψ [83]. Moreover, the effect of non-perturbative dynamics is neglected, the size of which is hard to estimate. Therefore, people may question the validity of the 12 % rule as a serious benchmark for comparing experimental data.

B. Experiment Estimation (I)

The first experimental estimation is based on the assumption that the decays of J/ψ and ψ' in the lowest order of QCD are classified into hadronic decays (ggg), electromagnetic decays (γ^*), radiative decays into light hadrons (γgg), and transition to lower mass charmonium states ($c\bar{c}X$) [72, 81]. Thus, using the relation $\mathcal{B}(ggg) + \mathcal{B}(\gamma gg) + \mathcal{B}(\gamma^*) + \mathcal{B}(c\bar{c}X) = 1$, one can derive $\mathcal{B}(ggg) + \mathcal{B}(\gamma gg)$ by subtracting $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\bar{c}X)$ from unity.

The calculated values of $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\bar{c}X)$, together with the values used to calculate them are summarized in Table III. As regards to ψ', two final states $\gamma\eta(2S)$ and $h_c(1^1P_1) + X$ with faint branching fractions are neglected in our calculation. By deducting the contributions $\mathcal{B}(\gamma^*)$ and $\mathcal{B}(c\bar{c}X)$, we find that $\mathcal{B}(J/\psi \to ggg) + \mathcal{B}(J/\psi \to \gamma gg) = (73.3 \pm 0.5)\%$ and $\mathcal{B}(\psi' \to ggg) + \mathcal{B}(\psi' \to \gamma gg) = (18.9 \pm 1.3)\%$, then the ratio of them is

$$Q_g = \frac{\mathcal{B}(\psi' \to ggg + \gamma gg)}{\mathcal{B}(J/\psi \to ggg + \gamma gg)} = (25.7 \pm 1.7)\% . \quad (15)$$

The above estimation is consistent with the previous ones [69, 81]. The relation between the decay rates of ggg and γgg is readily calculated in pQCD to the first order as [82]

$$\frac{\Gamma(J/\psi \to \gamma gg)}{\Gamma(J/\psi \to ggg)} = \frac{16}{5} \frac{\alpha}{\alpha_s(m_c)} \left(1 - 2.9 \frac{\alpha_s}{\pi}\right).$$

Using $\alpha_s(m_c) = 0.28$, one can estimate the ratio to be 0.062. A similar relation can be deduced for the ψ' decays. So we obtain $\mathcal{B}(J/\psi \to ggg) \simeq (69.0 \pm 0.5)\%$ and $\mathcal{B}(\psi' \to ggg) \simeq (17.8 \pm 1.2)\%$, while the "25.7% ratio"

TABLE III: Experimental data on the branching fractions for J/ψ and ψ' decays through virtual photon and to lower mass charmonium states used in this analysis. Most of the data are taken from PDG [27], except for $\mathcal{B}(J/\psi, \psi' \to \gamma^* \to \text{hadrons})$, which are calculated by the product $R \cdot \mathcal{B}(J/\psi, \psi' \to \mu^+\mu^-)$, with $R = 2.28 \pm 0.04$ [84]. In estimating the errors of the sums, the correlations between the channels are considered [85].

Channel	$\mathcal{B}(J/\psi)$	$\mathcal{B}(\psi')$
$\gamma^* \to$ hadrons	$(13.50\pm0.30)\%$	$(1.66\pm0.18)\%$
e^+e^-	$(5.94\pm0.06)\%$	$(7.35\pm0.18)\times10^{-3}$
$\mu^+\mu^-$	$(5.93\pm0.06)\%$	$(7.3\pm0.8)\times10^{-3}$
$\tau^+\tau^-$	$-$	$(2.8\pm0.7)\times10^{-3}$
$\gamma^* \to X$	$(25.37\pm0.35)\%$	$(3.41\pm0.27)\%$
$\gamma\eta_c$	$(1.3\pm0.4)\%$	$(2.6\pm0.4)\times10^{-3}$
$\pi^+\pi^- J/\psi$		$(31.8\pm0.6)\%$
$\pi^0\pi^0 J/\psi$		$(16.46\pm0.35)\%$
$\eta J/\psi$		$(3.09\pm0.08)\%$
$\pi^0 J/\psi$		$(1.26\pm0.13)\times10^{-3}$
$\gamma\chi_{c0}$		$(9.2\pm0.4)\%$
$\gamma\chi_{c1}$		$(8.7\pm0.4)\%$
$\gamma\chi_{c2}$		$(8.1\pm0.4)\%$
$c\bar{c}X$	$(1.3\pm0.4)\%$	$(77.7\pm1.2)\%$

in Eq. (15) stands well for both ggg and γgg. Although Q_g is considerably enhanced relative to Q_h in Eq. (1), it is fairly compatible with the ratios for the K^+K^- and $K^0_S K^0_L$ decay modes between ψ' and J/ψ, which are

$$\begin{aligned} Q_{K^+K^-} &= (26.6 \pm 4.5)\% \text{ (CLEO)}, \\ Q_{K^0_S K^0_L} &= (28.8 \pm 3.7)\% \text{ (BES)}, \\ Q_{K^0_S K^0_L} &= (32.2 \pm 5.2)\% \text{ (CLEO)}, \end{aligned}$$

according to the recent results from CLEO and BES [5–7]. The relation in Eq. (15) was discussed in the literature as the hadronic excess in ψ' decays [69, 81]. It implicates that while some modes are suppressed in ψ' decays, the dominant part of ψ' through ggg decays is enhanced relative to the 12% rule prediction in the light of J/ψ decays.

C. Experiment Estimation (II)

The second approach for estimating Q_h is to use the data on branching fractions for hadronic decays in final states containing pions, kaons, and protons that have already been measured for both the J/ψ and the ψ'. They are $\pi^+\pi^-$, K^+K^-, $p\bar{p}$, $\pi^+\pi^-\pi^0$, $p\bar{p}\pi^0$, $2(\pi^+\pi^-)$, $3(\pi^+\pi^-)$, $2(\pi^+\pi^-)\pi^0$, $3(\pi^+\pi^-)\pi^0$, $2(\pi^+\pi^-\pi^0)$, $2(K^+K^-)$, $K^+K^-\pi^+\pi^-$, $K^+K^-\pi^+\pi^-\pi^0$, $K^+K^-2(\pi^+\pi^-)$, $\pi^+\pi^- p\bar{p}$, $p\bar{p}\pi^+\pi^-\pi^0$ and so forth. Using the data compiled in Table IV, we have

$$\sum_{i=1}^{11} \mathcal{B}(J/\psi \to f_i) = (14.46 \pm 0.80)\%$$

and

$$\sum_{i=1}^{11} \mathcal{B}(\psi' \to f_i) = (1.70 \pm 0.23)\% .$$

It follows that

$$\begin{aligned} Q_s &= \sum_{i=1}^{11} \mathcal{B}(\psi' \to f_i) \Big/ \sum_{i=1}^{11} \mathcal{B}(J/\psi \to f_i) \\ &= (11.8 \pm 1.7)\% . \end{aligned} \quad (16)$$

TABLE IV: Branching fractions for ψ' and J/ψ decays, and Q_h values are also calculated. For $\psi' \to \pi^+\pi^-$, K^+K^- decays, the results are the weighted average of measurements from CLEO [5] and BES [86], while for $J/\psi \to \pi^+\pi^-\pi^0$, the result is the weighted average of measurements from BES [3] and BABAR [4]. Except for values with †, all others from PDG [27].

Final state	$\mathcal{B}_{J/\psi}(10^{-3})$	$\mathcal{B}_{\psi(2S)}(10^{-4})$	Q_h (%)
$\pi^+\pi^-$	0.147 ± 0.023	0.08 ± 0.05†	5.6 ± 3.5
K^+K^-	0.237 ± 0.031	0.63 ± 0.07†	26.5 ± 4.5
$p\bar{p}$	2.17 ± 0.08	2.65 ± 0.22	12.2 ± 1.1
$\pi^+\pi^-\pi^0$	21.2 ± 1.01†	1.68 ± 0.26	0.79 ± 0.13
$p\bar{p}\pi^0$	1.09 ± 0.09	1.33 ± 0.17	12.2 ± 1.9
$2(\pi^+\pi^-)$	3.55 ± 0.23	2.4 ± 0.6	6.8 ± 1.8
$3(\pi^+\pi^-)$	4.3 ± 0.4	3.5 ± 2.0	8.1 ± 4.7
$3(\pi^+\pi^-)\pi^0$	29 ± 6	35 ± 16	12.1 ± 6.1
$2(\pi^+\pi^-)\pi^0$	33.7 ± 2.6	26.6 ± 2.9	7.9 ± 1.1
$2(\pi^+\pi^-\pi^0)$	16.2 ± 2.1	45 ± 14	27.8 ± 9.4
$2(K^+K^-)$	0.78 ± 0.14	0.60 ± 0.14	7.7 ± 2.3
$K^+K^-\pi^+\pi^-$	7.2 ± 2.3	7.2 ± 0.5	10.0 ± 3.3
$K^+K^-\pi^+\pi^-\pi^0$	12.0 ± 3.0	12.4 ± 1.0	10.3 ± 2.7
$K^+K^-2(\pi^+\pi^-)$	4.7 ± 0.7	18 ± 9	38.3 ± 20.0
$p\bar{p}\pi^+\pi^-$	6.0 ± 0.5	6.0 ± 0.4	10.0 ± 1.1
$p\bar{p}\pi^+\pi^-\pi^0$	2.3 ± 0.9	7.3 ± 0.7	31.7 ± 12.8

Some remarks are in order here. First, we know that most of the multihadron final states in fact include sums of several two-body intermediate states, so the Q_s is not the exact ratio of ψ' to J/ψ inclusive hadronic decay rates, but the ratio on average of the exclusive decay channels as measured to date. In another word, Q_s represents a mixed effect which may deviate noticeably from the component Q-values. For example, the decay $\psi' \to \pi^+\pi^- K^+K^-$ can proceed through intermediate state $K^*(892)^0\overline{K}^*_2(1430)^0 + c.c.$, whose $Q = (2.9 \pm 1.3)\%$, is greatly suppressed [19] comparing with $Q = (10.0 \pm 3.3)\%$. Second, we notice that the results obtained by two estimations vary considerably; furthermore, by virtue of Table IV, it is obvious that many Q-values deivate from Q_s significantly while on average, Q_s is similar to pQCD Q_h. It would be an intriguing problem that with more and more data and higher accurate measurements if Q_s and Q_g could approximate with each other and be consistent with Q_h, or if Q_s and Q_g still deviate from each other prominently.

IV. COMMENT ON THE $\rho\pi$ PUZZLE

A. Theory Aspect

For all explanations involving $\rho\pi$ puzzle, there is a basic assumption that the non-relativistic potential model delineates the physics of charmonium decays to a good approximation. However, this assumption indeed requires examination in detail. First, as we have noticed in subsection III A, several corrections, should be added for the decay ratio between ψ' and J/ψ.

Second, we see the effect of non-perturbative dynamics is neglected, which is crucial for charmonium decays. Actually from various explanations of $\rho\pi$ puzzle, or more generally from the phenomenological explanation involving pQCD, we note that certain non-perturbative effect or nonlinear effect must be incorporated one way or the other in order to recount the experimental data. But this kind of effect could hardly been included in the present non-relativistic potential model.

Third, if the S-D wave mixing scenario holds as the solution of $\rho\pi$ puzzle, then the matrix element of D wave to light hadrons would be very large, which can hardly be accommodated in the potential model, or any other current theory. It indicates that the current understanding of charmonium decays may not be complete.

We envisage that new development of the theory should take into account the following features involving charmonium decays

- the mass effect for different charmonium states, such as J/ψ, ψ', ψ'' and so on;

- the non-perturbative or non-linear effect on the resonance decays;

- reasonable description for the known features of charmonium spectroscopy;

- quantitative consistence (with reasonable high accuracy) with the existing experimental measurements.

Herein it is also important to distinguish the quarkonium states (theoretical states) and mass eigenstates (physical states) [51, 53]. From a fairly theoretical point of view, if both states, say, J/ψ and ψ' did dominantly annihilate into three gluons, they would mix and could thus not be the putative quarkonium states. In a non-relativistic potential model, for example, the ψ' is simply a radial excitation of the J/ψ. This is a well defined picture in which J/ψ and ψ' are orthogonal states. If the annihilation into three gluons could be treated as a "perturbation" to the non-relativistic potential, then clearly the unperturbed states would mix and rearrange themselves into orthogonal mass eigenstates. The QCD dynamics may be such that the physical states, presumably mixtures of the theoretical $c\bar{c}$ states ($n^{2S+1}L_J$), are so built up that one of them strongly annihilates into three

perturbative gluons while the other does not. Mixing of the 1^3S_1 and 2^3S_1 states via three perturbative gluons has little effect on the charmonium mass spectrum, but may be crucial for the decay pattern.

B. Phenomenology Aspect

One may remember that at the early stage of the discovery of a narrow state J/ψ, the $c\bar{c}$ system was hailed as the Hydrogen atom of Quantum Chromodynamics, with the implied hope that the study of the newly discovered system could shed as much light on the dynamics of quark-antiquark interactions as the study of the Hydrogen atom had on Quantum Electrodynamics. But one may also notice the historical fact, even before Bohr's theory, Balmer series had been discovered for long, and the famous Rydberg formula had also been proposed, which laid a solid foundation for further theoretical improvement. If we are conscious of the more complicatedness of charmonia system comparing with Hydrogen atom, we may prepare for more hard and meticulous works. As a first step, it is necessary to develop a reliable and extensively applicable phenomenological model (PM).

The advantage of PM lies in that a well-defined PM contains few experimentally determined parameters which have clear physical meaning; moreover, with only few parameters determined from experiment, PM could produce concrete results which can be directly confirmed or falsified by experiment and may guid experimental searches. Such a model has a good relation with elementary principle of the theory, and if correct, can be used for further theoretical refinement. This point is noteworthy for the time being, since the general QCD can hardly provide solutions for special problems; conversely, we have to establish certain effective empirical model to advance our understanding for generic QCD principle.

Here we would like to mention few ideas of PM, which have or intend to provide a general description for charmonium decay.

1. Mixing Model

As we have noticed in subsection II C 4, whatever CI model or FK model, they could yield definite predictions for experimental test, and therefore provide clues for further development. In addition, according to Haber and Perrier's parameterization scheme [77], we could get the decay rate relations between different channels, and the proportions of different interaction amplitudes, all of which are useful information to understand the dynamics of charmonium decays.

The special feature of Rosner's S- and D-wave mixing scheme [56] is that it is simple, and it works both for suppressed and enhanced decay modes, and moreover, it

connects the J/ψ, ψ' and ψ'' decays together, and give predictions with little uncertainty.

2. Effective FSI

Miller once discussed the connection between the strong-coupling approximation to quantum chromodynamics and nuclear properties observed at low and medium energies and momentum transfer [87]. He suggested that the strong-coupling (corresponds to long-range interaction) version of QCD does reproduce the salient feature of the meson-baryon picture of low momentum transfer in nuclear physics. The derivation of nuclear physics indicates that quark aspects of ordinary nuclei are hidden in the hadronic degrees of freedom. Reversely, one may imagine that a quasi-meson or quasi-baryon structure could be formed in non-perturbative hadronization process. The meson-baryon picture in nuclear physics could be utilized, with some modification, as an effective FSI theory, or molecular-model theory. The recent molecular interpretations [88, 89] for the newly found state $Y(4260)$ could be treated as such an effective FSI theory.

Furthermore, the quasi-particles could even be real particles, and then with the residual strong force between the quarks inside the quasi-particles, some multiplets can be formed as the quasi-particle and anti-quasi-particle bound states. This idea was first put forth to explain a lots of low mass baryon-antibayron enhancements newly found by CLEO, Belle and BaBar collaborations [90].

3. Glueball and Hybrid

We have seen that the J/ψ-glueball mixing scheme is the first explanation proposed to explain the $\rho\pi$ puzzle. With the implication from lattice calculation [91], Suzuki once proposed a glueball-ψ' mixing scheme to explain the excess hadronic decays at the ψ' [92]. In fact, gluodynamics is always a tantalizing domain for theorists. Recently, some lattice evaluations suggest [93, 94] that the masses of some hybrid states could be low enough to be degenerated with charmonium states, such as J/ψ and/or χ_{cJ}. Unlike Glueball, these $c\bar{c}g$ hybrid states could be very narrow, so it may be very stimulating and noteworthy to search for and confirm these kinds of states.

C. Experiment Aspect

Physics is a science of experiment. Physical facts are bases for theoretical development and also criteria for checking theoretical hypotheses. It is a prominent fact that the early analyses on $\rho\pi$ puzzle based on meager experimental data often lead to unsatisfactory, sometimes premature, inferences, which were washed out easily by later accurate data. In fact, the current knowledge concerning the ψ' decays from experiments is still rather limited, even summing all charmless channels presented in PDG [27], the total branching fraction is less 2%. Such situation prevents us from laying down a solid foundation for elementary dynamics exploration. However, the estimation of Q value, as discussed in Section III, affords us some clues concerning the exploration of charmonium decay dynamics. Since many suppressed channels have been found, especially those such as $\rho\pi$ which is greatly suppressed in ψ' decay, and if the Q_h really represents the averaged value of inclusive hadronic decay, the estimation of Q_g indicates that either lots of enhanced decays are not discovered, or some particular decays only present in ψ', or both cases exist. Therefore, systematic experimental study of ψ' decays is anxiously awaited.

Moreover, study should be carried out not only on the ratios of ψ' to J/ψ decays, but also on the other ratios such as those between η_c' and η_c [79], and those between ψ'' and J/ψ[50], and/or many other ratios between different resonances for the same channel or between different channels about the same resonance [54]. All these studies will shed light on the understanding of charmonium decays. From our point of view, the progress would be more likely obtained from the analysis of the new experimental results, rather than from the inspiration of general theoretical principle.

V. SUMMARY

In this paper, we present a general review on the study of the $\rho\pi$ puzzle. Although there is still no satisfactory explanation for all existing experiment results, some enlightenment ideas have been put forth. In addition, we also discuss three methods of estimating the ratio of the branching fractions in J/ψ and ψ' decays. In the light of the present theoretical and experimental status, we argue that it is important to explore the potential models from a new point of view, it is necessary to search for and/or construct an effective phenomenological model, and it is especially crucial to perform systematic measurements of various charmonium decays.

As we know physics in the charm energy region is in the boundary domain between perturbative and nonperturbative QCD. Recently the observed hadronic decays of charmonium may give new challenges to the present theoretical understanding of the decay mechanisms. Many of the new charmonium states observed by Belle and BaBar which can hardly be accommodated by potential models have led to new theoretical speculations about charmonium spectroscopy and novel production mechanism [95].

Experimentally, the expected large data sample from CLEOc in the near future, and even larger sample from forthcoming detector BESIII will open for us a new era for charmonium dynamics study, even though we may not obtain a throughly revolutionary theory, we could acquire more profound understanding for the existing theory, at

the same time we could expect a brand-new comprehension for the $\rho\pi$ puzzle.

[1] T. Appelquist and H. D. Politzer, Phys. Rev. Lett. **34**, 43 (1975); A. De Rújula and S. L. Glashow, Phys. Rev. Lett. **34**, 46 (1975).

[2] Mark-II Collaboration, M. E. B. Franklin *et al.*, Phys. Rev. Lett. **51**, 963 (1983).

[3] BES Collaboration, J. Z. Bai, *et al.*, Phys. Rev. D **70**, 012005 (2004).

[4] Babar Collaboration, B. Aubert, *et al.*, Phys. Rev. D **70**, 072004 (2004).

[5] CLEO Collaboration, S. Dobbs *et al.*, Phys. Rev. Lett. **74**, 011150 (2006).

[6] BES Collaboration, J. Z. Bai *et al.*, Phys. Rev. D **69**, 012003 (2004).

[7] BES Collaboration, J. Z. Bai *et al.*, Phys. Rev. Lett. **91**, 052001 (2004).

[8] BES Collaboration, J. Z. Bai, *et al.*, Phys. Rev. D **54**, 1221 (1996); Erratum: ibid. **57**, 3187 (1998).

[9] BES Collaboration, M. Ablikim *et al.*, Phys. Rev. D **70**, 112003 (2004).

[10] BES Collaboration, M. Ablikim *et al.*, Phys. Rev. D **70**, 112007 (2004).

[11] BES Collaboration, M. Ablikim *et al.*, Phys. Lett. B **614**, 37 (2005).

[12] BES Collaboration, M. Ablikim *et al.*, Phys. Rev. D **70**, 077101 (2004).

[13] BES Collaboration, M. Ablikim *et al.*, Phys. Rev. D **72**, 072007 (2005).

[14] BES Collaboration, M. Ablikim *et al.*, Phys. Lett. B **619**, 247 (2005).

[15] BES Collaboration, M. Ablikim *et al.*, Phys. Rev. D **71**, 072006 (2005).

[16] CLEO Collaboration, N. E. Adam *et al.*, Phys. Rev. Lett. **94**, 012005 (2005).

[17] BES Collaboration, J. Z. Bai *et al.*, Phys. Rev. Lett. **81**, 5080 (1998).

[18] BES Collaboration, J. Z. Bai *et al.*, Phys. Rev. D **67**, 052002 (2003).

[19] BES Collaboration, J. Z. Bai *et al.*, Phys. Rev. D **69**, 072001 (2004).

[20] Z. Metreveli (for CLEO Collaboration), "Recent $\psi(2S)$ results from CLEOc", talk at Charm2006, Beijing, China, June 5-7, 2006.

[21] CLEO Collaboration, R. A. Briere *et al.*, Phys. Rev. Lett. **95**, 062001 (2005).

[22] CLEO Collaboration, G. S. Adams *et al.*, Phys. Rev. D **73**, 012002 (2006).

[23] CLEO Collaboration, D. Cronin-Hennessy *et al.*, hep-ex/0603026.

[24] S. J. Brodsky, G. P. Lepage, Phys. Rev. D **24**, 2848 (1981).

[25] P.G.O. Freund and Y. Nambu, Phys. Rev. Lett. **34**, 569 (1983).

[26] S. Okubo, Phys. Lett. **5**, 165 (1963); G. Zweig, CERN-preprint CERN-TH-401, 402, 412 (1964); Iizuka, Prog. Theor. Phys. Suppl. **37-38**, 21 (1996).

[27] Particle Data Group, W.-M. Yao *et al.*, J. Phys. **G 33** (2006) 1.

[28] W. S. Hou and A. Soni, Phys. Rev. Lett. **50**, 569 (1983).

[29] W. S. Hou and A. Soni, Phys. Rev. D **29**, 101 (1984).

[30] S. J. Brodsky, G. P. Lepage and S. F. Tuan, Phys. Rev. Lett. **59**, 621 (1987).

[31] Particle Data Group, M. Aguilar-Benitez *et al.*, Phys. Lett. B **170**, 1 (1986).

[32] C. T. Chan and W. S. Hou, Nucl. Phys. A **675**, 367c (2000).

[33] Y. H. Xie, "Searching for new particles with $J = 1$ in $\psi(2S)$ decays", Ph.D. thesis, IHEP, Chinese Academy of Sciences, 1998 (in Chinese).

[34] F. A. Harris, hep-ex/9903036 (unpublished).

[35] W. S. Hou, Phys. Rev. D **55**, 6952 (1997).

[36] Y. Q. Chen and E. Braaten, Phys. Rev. Lett. **80**, 5060 (1998).

[37] See, for example, M. Peardon, Nucl. Phys. Proc. Suppl. **63**, 22-27, (1998).

[38] M. Suzuki, Phys. Rev. D **57**, 5171 (1998).

[39] M. Anselmino, M. Genovese, and E. Predazzi, Phys. Rev. D **44**, 1597 (1991); M. Anselmino, M. Genovese, and D. E. Kharzeev, ibid. **50**, 595 (1994).

[40] M. Chaichian and N. A. Törnqvist, Phys. Lett. B **323**, 75 (1989).

[41] S. J. Brodsky and M. Karliner, Phys. Rev. Lett. **78**, 4682 (1997).

[42] V. L. Chernyak, hep-ph/9906387.

[43] G. Karl and W. Roberts, Phys. Lett. B **144**, 263 (1984).

[44] G. Karl and S-F. Tuan, Phys. Rev. D **34**, 1692 (1986).

[45] S. S. Pinsky, Phys. Lett. B **236**, 479 (1990).

[46] S. S. Pinsky, Phys. Rev. D **31**, 1753 (1985).

[47] K.-T. Chao, Proc. 17th Intern. Symp. on Lepton-Photon Interactions at high Energies, Beijing, China, 1995, ed. Z. P. Zheng and H. S. Chen (World Scientific, Singapore, 1996) p. 106.

[48] Y. S. Zhu, in *Proceedings of the 28th International Conference on High Energy Physics*, edited by Z. Adjuk and A. K. Wroblewski (World Scientific, Singapore, 1997).

[49] A. Bramon, R. Escribano, and M. D. Scadron, Phys. Lett. B **403**, 339 (1997).

[50] C. Z. Yuan, HEP & NP **30**, 488 (2006).

[51] J. M. Gérard and J. Weyers, Phys. Lett. B **462**, 324 (1999).

[52] P. Wang, X. H. Mo and C. Z. Yuan, Phys. Lett. B **557**,

197 (2003).

[53] P. Artoisenet, J. M. Gérard and J. Weyers, Phys. Lett. B **628**, 211 (2005).

[54] V. L. Chernyak and A. R. Zhitnitsky, Phys. Rep. **112**, 173 (1984).

[55] We did not consider the possible different mixing angle extracted from the Belle results by K. Y. Liu and K. T. Chao, Phys. Rev. D **70**, 094001 (2004).

[56] J. L. Rosner, Phys. Rev. D **64**, 094002 (2001).

[57] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. **D17** (1978) 3090; **D21** (1980) 313(E); **D21** (1980) 203(E).

[58] K. Heikkilä, N. A. Törnqvist and S. Ono, Phys. Rev. D **29**, 110 (1984); *ibid.* D **29**, 2136(E) (1984).

[59] Y.-P. Kuang and T. M. Yan, Phys. Rev. D **41**, 155 (1990).

[60] P. Wang, C. Z. Yuan and X. H. Mo, Phys. Lett. B **574**, 41 (2004).

[61] P. Wang, C. Z. Yuan and X. H. Mo, Phys. Rev. D **69**, 057502 (2004).

[62] P. Wang, X. H. Mo and C. Z. Yuan, Int. J. Mod. Phys. A **21**, 5163 (2006).

[63] P. Wang, X. H. Mo and C. Z. Yuan, Phys. Rev. D **70**, 077505 (2004).

[64] P. Wang, C. Z. Yuan and X. H. Mo, Phys. Rev. D **70**, 114014 (2004).

[65] CLEO Collaboration, D. Besson *et al.*, CLEO 05-27 (2005).

[66] X. Q. Li, D. V. Bugg and B. S. Zou, Phys. Rev. D **55**, 1421 (1997).

[67] S. F. Tuan, Commun. Theor. Phys. **33**, 285 (2000).

[68] M. Suzuki, Phys. Rev. D **60**, 051501 (1999).

[69] M. Suzuki, Phys. Rev. D **63**, 054021 (2001).

[70] J. Jousset *et al.*, Phys. Rev. D **41**, 138 (1990); D. Coffman *et al.*, Phys. Rev. D **38**, 2695 (1988); A. Bramon, R. Escribano and M.D. Scadron, Phys. Lett. B **403**, 339 (1997); M. Suzuki, Phys. Rev. D **58**, 111504 (1998); N.N. Achasov, Relative Phase between the Three Gluon and One Photon Amplitudes of the J/ψ Decays, in Proc. ninth International Conference on Hadron Spectroscopy (Protvino, Russia, August 2001), AIP Conf. Proc. **619**

649 (AIP, New York, 2002).

[71] G. López Castro *et al.*, Tests of Flavor Symmetry in J/ψ Decays, in: Proc. CAM-94 (ed. A. Zepeda, Cancum, Mexico, 1994), AIP Conf. Proc. **342**, 441 (AIP, New York, 1995).

[72] L. Köpke and N. Wermes, Phys. Rep. **174**, 67 (1989).

[73] R. Baldini, *et al.* Phys. Lett. B **444**, 111 (1998).

[74] C. Z. Yuan, P. Wang and X. H. Mo, Phys. Lett. B **567**, 73 (2003).

[75] J. P. Ma, Phys. Rev. D **65**, 097506 (2002).

[76] L. J. Clavelli and G. W. Intemann, Phys. Rev. D **28**, 2767 (1983).

[77] H. E. Haber and J. Perrier, Phys. Rev. D **32**, 2961 (1985).

[78] A. Seiden, H. F-W. Sadrozinski, and H. E. Haber Phys. Rev. D **38**, 824 (1988).

[79] T. Feldmann and P. Kroll, Phys. Rev. D **62**, 074006 (2000).

[80] P. Hoyer and S. Peigné, Phys. Rev. D **61**, 031501 (2000).

[81] Y. F. Gu and X. H. Li, Phys. Rev. D **63**, 114019 (2001).

[82] W. Kwong, Phys. Rev. D **37**, 3210 (1988).

[83] H. C. Chiang et al., Phys. Lett. B **324**, 482 (1994).

[84] K. K. Seth, Phys. Rev. D **69**, 097503 (2004).

[85] X. H. Mo, HEP & NP **29**, 831 (2005).

[86] S. W. Ye, "Study of some VP and PP modes of ψ' decays", Ph.D. thesis, University of Science and Technology of China, 1997 (in Chinese).

[87] G.A. Miller, Phys. Rev. C **39**, 1563 (1999).

[88] X. Liu, X.Q. Zeng and X. Q. Li, Phys. Rev. D **72**, 054023 (2005).

[89] P. Wang, X. H. Mo and C. Z. Yuan, hep-ph/0512329.

[90] C. Z. Yuan, X. H. Mo and P. Wang, Phys. Lett. B **626**, 95 (2005).

[91] C. J. Morningstar and M. J. Peardon, Phys. Rev. D **60**, 034509 (1999).

[92] M. Suzuki, Phys. Rev. D **65**, 097507 (2002).

[93] T-W. Chiu, hep-lat/0512029.

[94] X-Q. Luo and Y. Liu, hep-lat/0512044.

[95] E.S. Swanson, Phys. Rep. **429**, 243 (2006).